

# Pion decay within the framework of Very Special Relativity

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We illustrate some new and novel effects which arise in decay and scattering experiments within the framework of Very Special Relativity (VSR) using the charged pion decay process as an example. It is assumed that there is a small violation of Lorentz invariance and the true symmetry group of nature is a subgroup called SIM(2). This symmetry group postulates the existence of a fundamental or preferred direction in space-time. We construct effective interaction terms which violate Lorentz invariance but respect SIM(2) and impose a limit on the corresponding coupling strength using the observed pion decay rate. We also predict a shift in the angular dependence of the decay products due to VSR. In particular we find that these no longer display azimuthal symmetry with respect to the momentum of the pion. Furthermore the azimuthal and polar angle distributions show time dependence with a period of a sidereal day. This time dependence provides us with a novel method to test VSR in future experiments.

## I. INTRODUCTION

Lorentz invariance is experimentally verified to a very high degree of accuracy. Nevertheless, it is interesting to consider models which postulate a small violation of this symmetry. In particular, many quantum gravity models predict breaking of Lorentz invariance at Planck scale energy ( $M_{Pl} \approx 10^{19} GeV$ ) [1]. It is rather interesting that the observational data already rules out most of these models, except those based on supersymmetry [2–4]. In such models, violation of Lorentz invariance is suppressed by the factor  $\frac{M_{SUSY}^2}{M_{Pl}^2}$  [3].

An alternative framework to implement violation of Lorentz invariance is provided by the Very Special Relativity (VSR) [5]. In this framework one postulates that only a subgroup, such as, T(2), E(2), HOM(2) and SIM(2), of the full Lorentz group remains preserved [5]. The generators of HOM(2), for example, are  $T_1 = K_x + J_y$ ,  $T_2 = K_y - J_x$  and  $K_z$  where  $\mathbf{J}$  and  $\mathbf{K}$  represent rotation and boost respectively, while those of SIM(2) are  $T_1$ ,  $T_2$ ,  $J_z$  and  $K_z$ . A theory which is invariant only under one of these subgroups but not the full Lorentz group, necessarily breaks the discrete symmetries P, T, CP (or CT). However the dispersion relations of particles remain unchanged. Hence several consequence of SR, such as frame invariance of the speed of light, time dilation and velocity addition remain preserved [5, 6]. This also implies that the standard high energy tests

of Lorentz violation (LV) are not applicable in this case. Hence the current experimental limits on LV are much weaker in the case of VSR.

It is useful to define a null vector

$$n^\mu = (1, 0, 0, 1). \quad (1)$$

This vector is invariant under the E(2) and T(2) transformations but not under HOM(2) and SIM(2). In this paper we shall primarily be interested in small violations of Lorentz invariance which preserve SIM(2). We shall implement this by using effective Lagrangian approach and construct interaction terms in terms of  $n^\mu$  which respect SIM(2) but violate Lorentz invariance. The vector  $n^\mu$  is given by Eq. 1 only in a particular reference frame. In general, the form of  $n^\mu$  would change under Lorentz transformations and rotations. However it is always possible to make a HOM(2) (and SIM(2)) transformation into the rest frame of a particle [5]. Hence we can choose a frame at rest with respect to the particle or to the laboratory in which  $n^\mu$  takes the form given in Eq. 1. However the orientation of the particle momentum relative to the z-axis in this frame has to be taken into account while making experimental predictions, as discussed below.

There has been considerable theoretical effort in order to understand the phenomenological implications of VSR [7–18]. In this paper we illustrate some new and novel tests of VSR [5] by considering charged pion decay as an example, using an effective action approach. Similar effects are likely arise in a wide range of decay and scattering processes within VSR. Charged pion decay has been studied to constrain the Lorentz violation (LV) in the weak sector [19, 20], However none of these

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studies have investigated this decay process within the framework of VSR.

The decay amplitude within the standard model can be computed by introducing the following effective interaction term:

$$\mathcal{L}_{\pi,SM} = V_{ud} \frac{G_f}{\sqrt{2}} f_\pi \partial_\mu \pi^- \bar{\psi}_l \gamma^\mu (1 - \gamma^5) \psi_\nu + h.c. \quad (2)$$

where  $\pi^-$ ,  $\psi_l$  and  $\psi_\nu$  represent the charged pion, charged lepton and the neutrino fields respectively. Here  $f_\pi = 132$  MeV is the pion decay constant,  $V_{ud} = \cos \theta_c$  is the CKM matrix element and  $\theta_c$  the Cabibbo angle. This leads to the standard formula for the weak differential decay rate of pions.

## II. VSR INVARIANT EFFECTIVE LAGRANGIAN

In this section we propose an effective Lagrangian which violates Lorentz invariance but respects VSR. A similar approach is taken to study the Lorentz violations at ultra high energies induced by quantum gravity effects. As mentioned in the Introduction, we can write VSR invariant fermion mass terms. Here we shall primarily be interested in the interaction terms which respect VSR but violate Lorentz invariance. The VSR invariant effective Lagrangian density for the coupling of pions with leptons can be written as

$$\mathcal{L}_\pi = \mathcal{L}_{\pi,SM} + g \left( \frac{n_\mu}{n \cdot \partial} \pi^- \right) \bar{\psi}_l \gamma^\mu (1 - \gamma^5) \psi_\nu + h.c. \quad (3)$$

where  $n^\mu$  is given by Eq. 1. The first term gives the standard decay amplitude for pion. The second term respects VSR but violates Lorentz invariance due to the presence of preferred axis  $n^\mu$ . The vector  $n^\mu$  is invariant under  $T_1$ ,  $T_2$ , rotation about z-axis  $J_z$  but transforms as  $n^\mu \rightarrow e^\epsilon n^\mu$  under boost along z-direction  $K_z$ . Here  $\epsilon$  is the transformation parameter. The non-local term  $\frac{n_\mu}{n \cdot \partial}$  is homogeneous in  $n^\mu$ . Hence the Lagrangian density is invariant under the SIM(2) subgroup [7].

In terms of the fundamental fields a LV, VSR invariant term in the effective action may arise in the electroweak interaction of quarks or leptons [9]. Alternatively we motivate it by introducing an effective coupling of pions with quarks using the linear sigma model at the quark level. Besides the standard terms we add a LV term given by,

$$\mathcal{L}_{\sigma,LV} = g_\sigma \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix} \left[ \frac{n_\mu}{n \cdot \partial} (\sigma + i\tau \cdot \pi \gamma^5) \right] \gamma^\mu \begin{pmatrix} u \\ d \end{pmatrix} \quad (4)$$

Here  $u$ ,  $d$ ,  $\sigma$  and  $\pi$  represent the up quark, down quark, scalar  $\sigma$  field and the pseudoscalar pion field respectively. The quarks in turn couple to the  $W$  bosons through the standard electroweak Lagrangian which further decay into leptons. Integrating over the quark loop leads to an effective vertex given by Eq. 3. We point out that besides the LV term in Eq. 3 there exist a wide range of terms [9] we can write down whose implications can be explored in future.

## III. PION DECAY IN THE REST FRAME

In this section we compute the decay rate of charged pion ( $\pi^-(q) \rightarrow \mu^-(p) + \bar{\nu}_\mu(k)$ ) in its rest frame within the VSR framework. As explained above, we can always make a SIM(2) transformation to the rest frame of a particle. Our action is invariant under this transformation, although the vector  $n^\mu$  changes by an overall constant. However the change cancels out in the amplitude. Here we work in a frame ( $S$ ) in which the vector  $n^\mu$  is given by Eq. 1 up to an overall constant. The LV contribution is assumed to arise entirely from the interaction term in Eq. 3. We point out that the VSR invariant quadratic terms do not change the dispersion relations [7]. Hence the kinematics of the incoming and outgoing particles remain unchanged. The dominant contribution to the differential decay rate arises due to standard model and LV interference term. This leads to a contribution proportional to  $g(1 + \cos \theta)$  where  $\theta$  is the angle between the muon three momentum and the z-axis in the fundamental frame  $S$ .

We next impose a limit on  $g$  by assuming that the standard observed value of the pion decay rate arises entirely from the Standard Model and demanding that the LV terms give a contribution less than the error in the observed value. The life time ( $\tau$ ) of the charged pion is  $(2.6033 \pm 0.0005) \times 10^{-8}$  s [21]. This leads to the limit,  $g < g_0$  where  $g_0 = 2.1 \times 10^{-12}$  GeV. The relative change of differential decay rate can be expressed as

$$\begin{aligned} \Delta &= \frac{\frac{d\Gamma}{d\Omega}|_{g \neq 0} - \frac{d\Gamma}{d\Omega}|_{g=0}}{\frac{d\Gamma}{d\Omega}|_{g=0}} \\ &= \frac{2\sqrt{2}g}{f_\pi m_\pi^2 G_f |V_{ud}|} [1 + \cos \theta] . \end{aligned} \quad (5)$$

Hence we find that even in the rest frame of pion, the muon distribution is not isotropic and depends on the polar angle  $\theta$  due to the LV contributions.

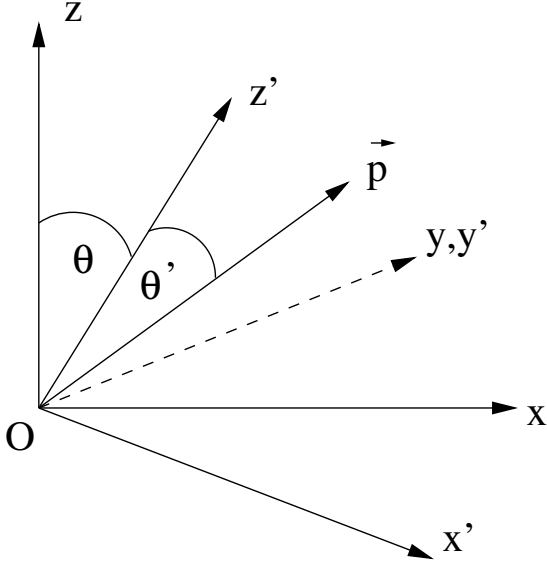


FIG. 1. Here  $z$  denotes the preferred axis and  $x, y$  some chosen coordinate axes. The beam direction is taken to be along the  $z'$ -axis which makes an angle  $\theta$  relative to  $z$ -axis. The axis  $x'$  is chosen to lie in the  $x-z$  plane. Hence the  $y$  and  $y'$  axes, pointing into the plane of the paper, coincide with one another. The momentum of the muon, denoted by  $\vec{p}$ , makes an angle  $\theta'$  relative to the  $z'$ -axis.

The dependence provides a qualitatively new test of LV theories which respect VSR.

#### IV. PION DECAY IN THE LABORATORY FRAME

In this section, we determine the differential decay rate assuming that pion has non-zero momentum in the laboratory frame. It is useful to define two frame  $S$  and  $S'$ , both at rest with respect to the laboratory. In frame  $S$ ,  $n^\mu$  is given by Eq. 1 up to an irrelevant overall constant. Let us now consider a beam of pions moving along the  $z'$  direction making an angle  $\theta$  with the preferred axis, as shown in Fig. 1. Let  $q, p$  and  $k$  denote the momenta of  $\pi^-$ ,  $\mu^-$  and  $\bar{\nu}$  respectively. Here  $xyz$  and  $x'y'z'$  refer to  $S$  and  $S'$  coordinate systems respectively. We have used rotational symmetry about the  $z$ -axis in the frame  $S$  in order to choose  $x$ -axis such that  $y'$  is aligned with the  $y$ -axis. Hence  $z'$  and  $x'$  lie in the  $x-z$  plane. The final state muon makes an angle  $\theta'$  w.r.t. the beam, i.e. the  $z'$  axis.

We find that the differential decay rate picks up a small correction to the  $\theta'$  dependence of the decay rate due to the LV term. Furthermore it induces a

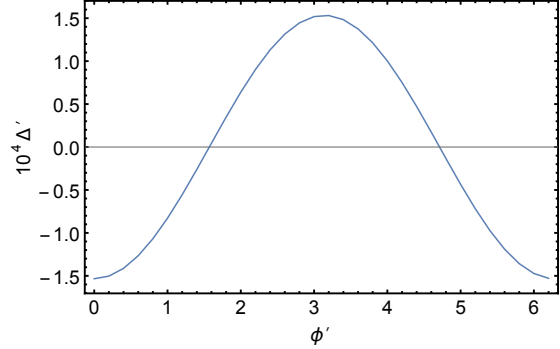


FIG. 2. The azimuthal angle  $\phi'$  dependence of the final state muon distribution. The observable  $\Delta'$  is defined in Eq. 6

$\phi'$  dependence of the final state muon distribution, which is absent in the standard model. The  $\phi'$  dependence of the decay rate can be quantified by defining

$$\Delta' = \frac{\frac{d\Gamma}{d\phi'} - \Gamma_{avg}}{\Gamma_{avg}} \quad (6)$$

where  $\Gamma_{avg} = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\Gamma}{d\phi'} d\phi'$  is the decay rate averaged over  $\phi'$ . In Fig.(2) we plot  $\Delta'$  as a function of  $\phi'$  for the choice of parameters, pion energy  $E = 200$  MeV and  $\theta = \pi/4$ . We see that the distribution peaks at  $\phi' = \pi$  and is minimum at  $\phi' = 0$ . From Fig. 1 we see our choice of coordinate system is such that the beam axis, i.e.  $z'$ , lies in the  $x'-z$  plane. Hence  $\phi'$  is the azimuthal angle in the  $x', y', z'$  coordinate system which is chosen such that  $z'$  lies in the  $x'-z$  plane

#### A. Daily Variation

The angle between the preferred axis and the beam direction is expected to change with time due to rotation of Earth. Due to this change the contribution to the differential decay rate arising from the LV term is expected to show periodic variation with a period of 1 sidereal day. Both the observables  $\Delta$  and  $\Delta'$  are expected to show time dependence. In particular we expect that the peak position of  $\Delta'$  as a function of the azimuthal angle  $\phi'$  in laboratory frame will show a periodic shift with time.

Let us assume that an observer is located at the latitude  $\lambda$ . We choose a local laboratory coordinate system at this location, denoted by  $x''y''z''$ . Here  $z''$  is the along the direction of the beam and

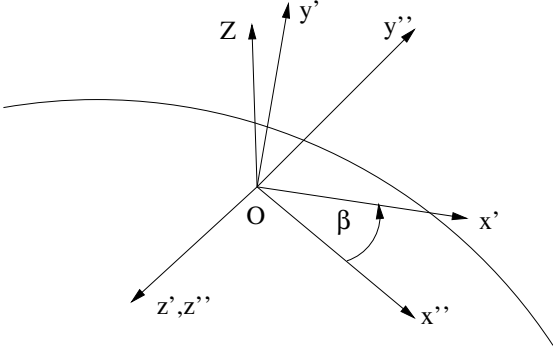


FIG. 3. The laboratory coordinates  $x''y''z''$  and the local coordinates  $x'y'z'$  at the position of the observer  $O$  located at latitude  $\lambda$ . Here  $z'$  and  $z''$  are along the beam direction. The rotation axis of Earth  $Z$  is also shown. The  $y''$  coordinate is taken to be the local normal, pointing upwards. The  $x'$  direction is chosen such that it lies in the  $z - z'$  frame as shown Fig. 1.

$y''$  is chosen along the local vertical. It is also convenient to define another local frame  $x'y'z'$  such that  $z'$  is along the beam direction, i.e. same as  $z''$  and  $x'$  lies in the  $z - z'$  plane. We denote the angle between  $z$  and  $z'$  by  $\theta$  as shown in Fig. 1. Hence  $\hat{z} \cdot \hat{z}' = \hat{z} \cdot \hat{z}'' = \cos \theta$ . The  $x'$ -axis lies in the same plane as  $z$  and  $z'$  (or  $z''$ ). The  $x' - y'$  and  $x'' - y''$  planes coincide and we denote the angle between  $x$  and  $x'$  as  $\beta$ . Using this we obtain

$$\hat{z} = \cos \theta \hat{z}'' - \sin \theta (\cos \beta \hat{x}'' + \sin \beta \hat{y}'') \quad (7)$$

The coordinates  $x'y'z'$  at any particular time are exactly the same as in Fig. 1. Hence once we obtain the angle  $\theta$ , which is time dependent, we can obtain the differential decay rate in this frame at any particular time using the formulism described earlier. In this frame the peak in the  $\phi'$  distribution occurs at  $\phi' = \pi$  as shown in Fig. 2. We next need to transform to the laboratory frame  $x''y''z''$ . This simply amounts to a rotation about the  $z'$  (or  $z''$ ) axis by an angle  $-\beta$ . Hence in this frame the peak occurs at  $\phi'' = \pi - \beta$ .

We next determine the time dependence of the angles  $\theta$  and  $\beta$  due to the rotation of Earth. We use the astronomical equatorial system as our fixed coordinate system denoted by  $XYZ$ . In this case the  $Z$ -axis is parallel to the rotation axis of Earth and the  $X - Y$  plane is same as the equatorial plane. Let us assume that the preferred axis  $z$  in this frame can be expressed as,

$$\hat{z} = \cos \theta_p \hat{Z} + \sin \theta_p (\cos \phi_p \hat{X} + \sin \phi_p \hat{Y}) \quad (8)$$

The axis  $y''$  makes an angle  $(\pi/2) - \lambda$  with respect to the  $Z$  axis at all times. At some initial time

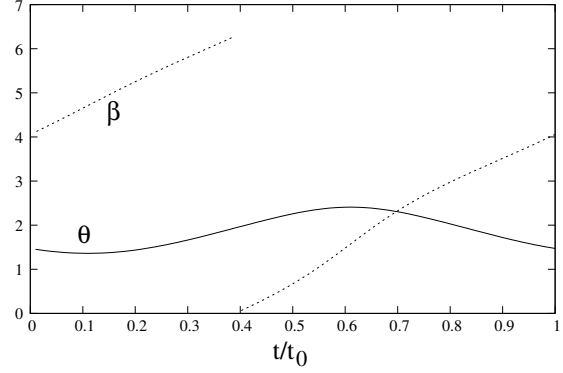


FIG. 4. The time dependence of  $\theta$  (solid curve) and  $\beta$  (dotted curve),  $0 \leq \beta < 2\pi$ , as a function of time. Here  $t_0$  is equal to 1 sidereal day. The observer is located at  $\lambda = 30^\circ$  and the remaining angles (in radians) are chosen as,  $\theta_p = 0.4\pi$ ,  $\phi_p = 0.3\pi$  and  $\alpha = 0.1\pi$ . The peak position in the  $\phi''$  distribution occurs at  $\pi - \beta$ .

$t = 0$  let the azimuthal angle of  $y''$  in this system be  $\alpha$ . Hence we can express the laboratory frame  $x''y''z''$  in terms of the fixed coordinate system as

$$\begin{aligned} \hat{y}'' &= \sin \lambda \hat{Z} + \cos \lambda (\cos \alpha \hat{X} + \sin \alpha \hat{Y}) \\ \hat{z}'' &= -\cos \lambda \hat{Z} + \sin \lambda (\cos \alpha \hat{X} + \sin \alpha \hat{Y}) \\ \hat{x}'' &= -\sin \alpha \hat{X} + \cos \alpha \hat{Y} \end{aligned} \quad (9)$$

At a later time  $t$  the same formulas hold with the angle  $\alpha$  replaced by  $\tilde{\alpha} = \alpha + \delta$ , where  $\delta = 2\pi t/t_0$  and  $t_0$  is equal to a sidereal day. Using this we can directly compute the angles  $\theta$  and  $\beta$  at any time by using  $\cos \theta = \hat{z} \cdot \hat{z}''$ ,  $\sin \theta \cos \beta = (\hat{z} \times \hat{z}'') \cdot \hat{y}''$  and  $\sin \theta \sin \beta = -(\hat{z} \times \hat{z}'') \cdot \hat{x}''$ . Here  $0 \leq \theta \leq \pi$  and  $0 \leq \beta < 2\pi$ . The time dependences of  $\theta$  and  $\beta$  are shown in Fig. 4 for a particular choice of parameters  $\lambda$ ,  $\theta_p$ ,  $\phi_p$  and  $\alpha$ .

The daily variation of differential decay rate provides a very interesting way to test the LV contribution due to VSR. We may divide each sidereal day into a chosen number of bins. The data in each bin can be accumulated over a large number of days in order to test for the daily variation in the peak position of the azimuthal ( $\phi''$ ) distribution. Correspondingly we can test the time dependence of the  $\theta'$  (or  $\theta''$ ) of the decay rate. Here  $\theta'$  (or  $\theta''$ ) is simply the angle of the muon momentum relative of the beam direction. In testing the angular dependence the main complication is the detector response, which may not be isotropic. However the detector response is not expected to be time dependent. Hence it can be taken out by subtracting out the time independent component in the  $\phi''$

and  $\theta'$  distributions.

## V. CONCLUSION

We have suggested a new way to test VSR by taking the charged pion decay process as an example. Due to the presence of a preferred direction in VSR, we find that final state muon distribution acquires an azimuthal angle dependence

relative to pion beam. Furthermore both the azimuthal and polar angle distributions acquire periodic time dependence with a period of one sidereal day. This time dependence provides us with an effective way to test the principle of VSR at future particle physics experiments. The phenomenon is not limited to pion decay but may be observed in many decay and scattering processes if VSR is the true symmetry of nature.

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